Using EKF tweaks with Sigma Point Kalman Filters

In Sigma Point Kalman Filters (SPKF, see **[Merwe2004]**) Weighted Statistical Linear Regression technique is used to approximate nonlinear process and measurement functions:

$$\mathbf{y} = g(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{b} + \mathbf{e},$$

$$\mathbf{P}_{ee} = \mathbf{P}_{yy} - \mathbf{A}\mathbf{P}_{xx}\mathbf{A}^{\top}$$

where:

$\mathbf{e}$ is an approximation error,

$$\mathbf{A} = \mathbf{P}_{xy}^{\top}\mathbf{P}_{xx}^{-1},$$

$$\mathbf{b} = \bar{\mathbf{y}} - \mathbf{A}\bar{\mathbf{x}},$$

$$\mathbf{P}_{xx} = \sum_i w_{ci}\left(\chi_i - \bar{\mathbf{x}}\right)\left(\chi_i - \bar{\mathbf{x}}\right),$$

$$\mathbf{P}_{yy} = \sum_i w_{ci}\left(\gamma_i - \bar{\mathbf{y}}\right)\left(\gamma_i - \bar{\mathbf{y}}\right),$$

$$\mathbf{P}_{xy} = \sum_i w_{ci}\left(\chi_i - \bar{\mathbf{x}}\right)\left(\gamma_i - \bar{\mathbf{y}}\right),$$

$$\gamma_i = g(\chi_i)$$

$$\bar{\mathbf{x}} = \sum_i w_{mi}\chi_i,$$

$$\bar{\mathbf{y}} = \sum_i w_{mi}\gamma_i,$$

w_{ci} are covariation weghts, w_{mi} are mean weights.

This means that approximation errors of measurements may be treated as a part of additive noise and so we show that in SPKF we can use the following approximation of $\mathbf{S}_k$:

$$\mathbf{S}_k = \mathbf{H}_k\mathbf{P}_{k|k-1}\mathbf{H}_k^{\top} + \tilde{\mathbf{R}}_k,$$

where

$$\mathbf{H}_k = \mathbf{P}_{xz,k}^{\top}\mathbf{P}_{xx,k}^{-1},$$

$$\tilde{\mathbf{R}}_k = \mathbf{R}_k + \mathbf{P}_{ee,k},$$

this enables us to use some EKF tricks such as adaptive correction or generalized linear models with SPKF.

References

[Merwe2004] R. van der Merwe, "Sigma-Point Kalman Filters for ProbabilisticInference in Dynamic State-Space Models", PhD Thesis, OGI School of Science & Engineering, Oregon Health & Science University, USA